Holographic Complexity in Non-Commutative Gauge Theory

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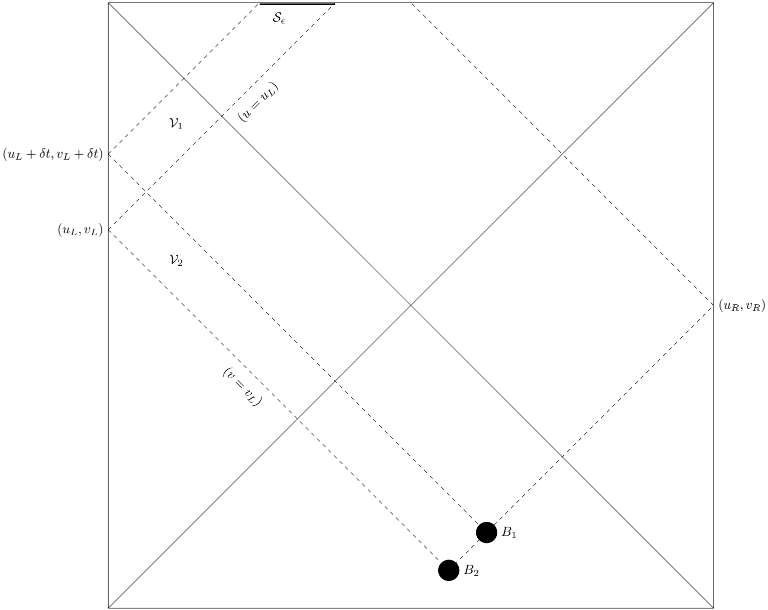
Quantum Circuit Complexity

- Consider a Hilbert space $\mathcal{H}$, e.g., the Hilbert space for N quantum bits.
- A universal gate set $\{g_i\}$ for $\mathcal{H}$ is a set of unitary operators on the Hilbert space such that any unitary U acting on $\mathcal{H}$ can be approximated by some product $\prod_i g_{\alpha_i}$ to within a small tolerance ϵ.
- Such a product of gates is referred to as a quantum circuit.
- The quantum circuit complexity of a unitary U is then the minimum number of gates needed to approximate U to within the tolerance.
- In the example of qubits, one typically considers gates that act on a single qubit or pairs of qubits at a time.
- Given some reference state $|\psi_R\rangle$, one may define the complexity of a state $|\psi\rangle$ as the minimum of complexity $C(U)$ over all unitaries U such that $|\psi\rangle = U|\psi_R\rangle$.

Holographic Complexity

- So far there are at least three proposals for the holographic dual of the complexity of the state of the boundary field theory. The proposed duals are the volume of a maximal spatial slice ('complexity = volume'), the action evaluated on a Wheeler-DeWitt (WDW) patch ('complexity = action'), and the spacetime volume of a WDW patch ('complexity = spacetime volume').
- These proposals are motivated by the late time behavior of the volume of a two-sided black hole
- All three proposals involve different measures of the 'size' of a black hole
- One piece of evidence for these conjectures is that they exhibit the correct behavior for precursor operators, which measure the difference between the state now, and what state we would have now had we inserted an operator $\mathcal{O}$ at an earlier time t.
- For this work we have focused on the complexity = action conjecture.

Action on a WDW patch



Non-Commutative SYM

- We would like to test complexity = action in a new context. However, computing the complexity of a state in a strongly couped field theory is not something we know how to do. Instead, we will rely on our intuition about the qualitative behavior of complexity in some novel situation.
- A good candidate is SYM living on a non-commutative geometry, i.e., a geometry where two of the spatial directions don't commute.
- A gravity dual to such a theory was derived in the late 90's by a number of authors [1, 2].
- Generalizations of this system to other numbers of dimensions have also been considered in, e.g., [3, 4].
- These solutions are obtained by considering a stack of Dp-branes in type II string theory and exciting a compenent of the NS-NS 2-form B-field along two of the worldsheet directions.
- Additional non-commutativities can be introduced by turning on additional complenets of the B-field.

Non-Commutativity and Complexity: A heuristic argument

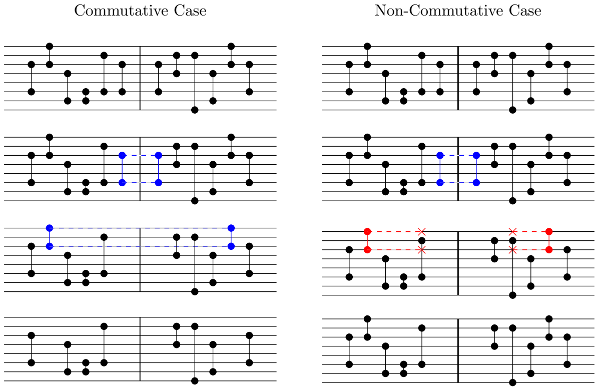


Figure : Consider an optimal circuit that evolves our state by a small time δt. As we compose this circuit many times, we expect some cancelations between gates in adjacent copies of the ciruit. Non-commutativity acts as an impediment to such cancelation by making fewer gates commute (due to non-locality), and so the final ciruit after cancelation is more complex.

Our Results

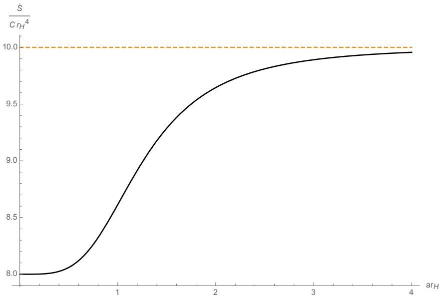


Figure : Late time action growth rate normalized by $C = \frac{\alpha^4 \Omega_5 V_3}{\hat{g}_s^2}$ and extra r_H dependence, versus ar_H, which is the Moyal scale measured in units of thermal length.

p	$m = 0$	$m = 1$	$m = 2$
2	12	12	-
3	8	10	-
4	5	5	8
5	4	5	6

Conclusions

- For $p = 3, 5$ we do see an increase, as expected.
- Though we did not see an increase for $p = 2$ or for $p = 4$ with a single non-trivial commutator, we did not see a decrease either.
- Overall, the results are consistent with the heuristic argument above.
- This result is in tension with the idea that commutative black holes are he fastest possible computers
- In future work we plan to repeat our calculations for complexity $=$ volume.

References

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