Holographic Complexity and Volume

Work with Stefan Eccles, Ted Jacobson, and Phuc Nguyen
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Introducion

We would like to study properties of maximal volume slice. Why?

- Conjectured by Susskind to be dual to circuit complexity of CFT state.
- Regardless of complexity, can capture growth of wormhole for eternal black hole.
- New tools brought to like by Freedman and Headrick, Headrick and Hubeny (max-cut/min-flow).

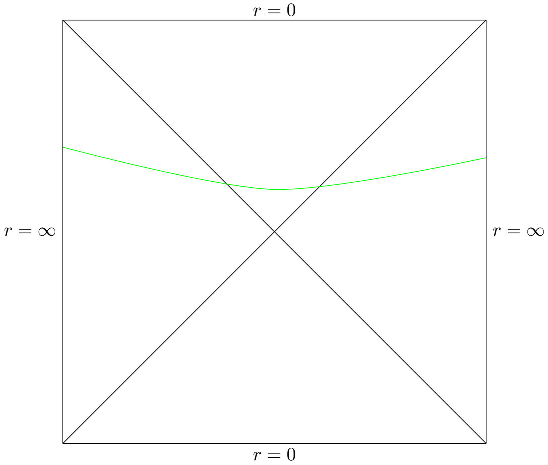

Figure : A maximal spatial slice

Max-Cut/Min-Flow

We are interested in applying ideas similar to the bit-thread ideas of Freedman and Headrick. This makes use of the min-cut/max-flow theorem, by which you can find a maximum flux in place of a minimal area. In Lorentzian signature, this becomes max-cut/min-flow as detailed by Headrick and Hubeny.

- Let a *flow* be a timelike future directed divergence free vector field v such that everywhere in spacetime, $1 \leq |v|$.
- Consider a Lorentzian manifold with a timelike boundary, and let σ be a cauchy slice of that boundary.
- Then the volume of the maximal slice of the bulk bounded by σ can be found by finding the minimum over all flows of the flux through *any* slice bounded by σ.
- max-cut/min-flow has a *nesting property*, namely that given two boundary cauchy slices σ_1 and σ_2 such that *sigma*$_1$ is entirely to the past of σ_2, then there is a flow (highly non-unique) flow whose flux computes the maximal volumes bounded by both σ_1 and σ_2.
- In fact, we could have taken an entire boundary foliation.

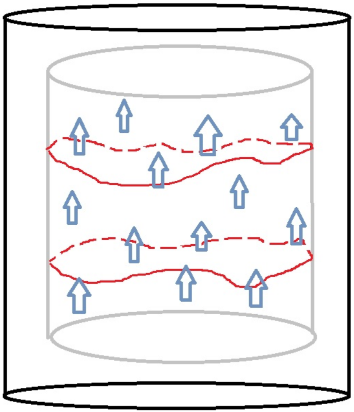

Figure : Volume Flow

Volume Flow

- The flows defined above are in general highly non-unique.
- However, whenever a boundary foliation induces a bulk foliation by maximal slices, the unit vector field normal to this bulk foliation is an example of such a flow.
- We will focus on this particular flow, which we will call the volume flow.
- Question: Does a foliation of the boundary automatically give a foliation of the bulk by maximal slices?
- In our paper, we answer yes given Einstein's equation and the strong energy condition.

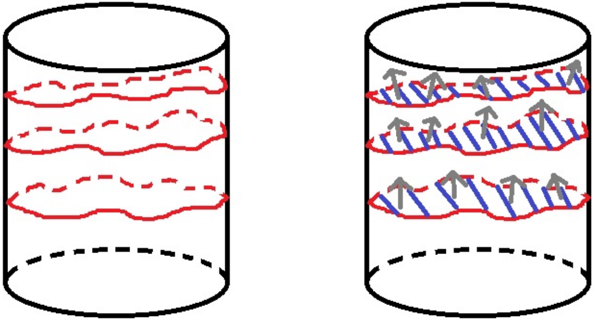

Figure : Foliations

Insights from Volume Flows

What do we learn from these volume flows?

- In looking at examples, we see that volume seems to flow away from boundary, possibly indicating a flow from UV into IR (of dual theory).
- In volume language, it is clear that the growth of the wormhole is generic:
  - Given any future horizon, volume flow can have an inbound flux but not outbound flux.
  - This is of course a direct consequence of the flow being timelike and future directed.
  - Perhaps reminiscent of the proposed 2nd law of complexity?
- We can also show a monotonicity property for the rate of increase of the maximal volume (in the boundary time).

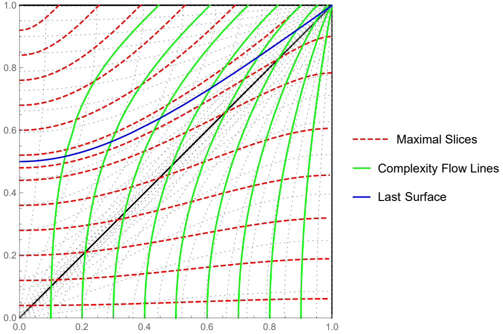

Figure : An example of a volume flow with a future horizon

A Monotonicity Property

- Consider boundary cauchy slices σ_1 and σ_2, and let their union be denoted by σ_U.
- Define σ_+ to be the set of points x in σ_U such that the future of x only intersects σ_U at x.
- Likewise define σ_- to be the set of points x in σ_U such that the past of x only intersects σ_U at x.
- Then $\text{Vol}(\sigma_1) + \text{Vol}(\sigma_2) \leq \text{Vol}(\sigma_+) + \text{Vol}(\sigma_-)$
- We can prove this based on the definition of a maximal slice without reference to flows, but there is a proof of this fact from flows based on their nesting property, which is exactly the Lorentzian analog of the proof of SSA in Freedman and Headrick.
- Applied to a two sided black hole, this implies that the rate of increase of maximal volume (as you increase the $t_L + t_R$, where both times increase towards the future) is monotonic.

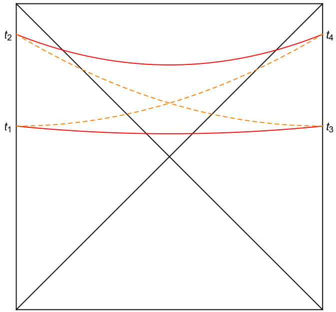

Figure : Maximal slices associated to different boundary slices for two-sided BH.

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Complexity = Action or Volume?

- Susskind's original conjecture that the maximal volume is dual to circuit complexity ('complexity = volume') has since been updated to a conjecture that the action evaluated on the causal development of such a slice is what is dual to circuit complexity ('complexity = action')
- In general the arguments that these conjectures are reasonable are broadly similar (switchback effect, scaling)
- Argument for replacing 'complexity = volume' by 'complexity = action' is largely due to the scaling.
  - Complexity is unitless, action and volume are unitful.
  - There is an expectation that the late time rate of change of complexity for thermal states should scale like TS.
  - For action, we can fix units by dividing by $\hbar$, and the scaling is already okay.
  - For volume, to get correct scaling, must defide by AdS radius for large black holes, but the horizon radius for small black holes.
  - We notice that you get the right scaling in 'complexity = volume' in a wide variety of cases if one divides by the maximal time from the horizon to the last slice.
  - On the other hand, this only works when there is a horizon.
- The rate of change of the action of the WDW patch is known not to increase monotonically, and instead reaches a local maximum before asyptotically approaching a smaller value from below (see Carmi et al.).
- Perhaps it is less clear that 'complexity = action' is an improvement on 'complexity = volume'.